NAIC-ID(RS)T-0011-96

NATIONAL AIR INTELLIGENCE CENTER

ALGORITHM RESEARCH ON ORBIT DETERMINATION FOR LOW ORBIT SATELLITES USING GPS

by

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HUMAN TRANSLATION

NAIC-ID(RS)T-0011-96 18 March 1996

MICROFICHE NR: 96C000247

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English pages: 16

Source: China Astronautics and Missilery Abstracts, Vol. 2, Nr. 4, 1995; pp. 22-31

Country of origin: China
Translated by: SCITRAN
F33657-87-D-0096
Requester: NAIC/TASS/Scott D. Feairheller
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ABSTRACT Detailed discussions are made of principles and methods associated with orbital determinations for low orbit spacecraft making use of GPS systems. Analysis is done of problems which should be considered with regard to low orbit spacecraft during selection of satellites. In conjunction with this, satellite selection tests are carried out on the basis of navigation satellite as well as user models. Discussions are made of making use of least square methods to make calculations to determine orbits. Detailed analysis is done of Kalman filter algorithms. With a view to low orbit spacecraft, a nine state Kalman wave filter model is put forward. In conjunction with this, simulation analog calculations are carried out on this foundation. Besides this, on the basis of special user requirements with regard to electric power source energies and reliability, one type of relatively practical orbit determination method is put forward. Simulation results clearly show that the method put forward satisfies user requirements in terms of accuracy and is appropriate for actual engineering applications.

SUBJECT TERMS Global positioning system Least square method Kalman flitering Satellite algorithm

The global positioning system (GPS) is a second generation satellite navigation system developed on the foundation of the TRANSIT satellite navigation system. At the present time, installation work on the whole 24 satellite navigation constellation is already completed. It is possible to realize contiuous, real time, all weather three dimensional positioning on a world scale. There are broad applications in such realms as land communications, geodetic surveys, map making, maritime navigation, aviation, and so on. The prospects for applications in the field of space are also very wide.

At the present time, there are basically the three types of methods below for determining spacecraft orbits [1]: ground equipment orbit determination method, tracking and data relay satellite system (TDRSS) orbit determination method, and the GPS orbit determination method. Looking at the situation of China, due to limitations associated with geographical breadth and economic conditions, making use of the first two types of methods has definite limitations in both cases. As a result, making use of GPS autonomous orbit determinations is one type of very good choice. This article discusses methods associated with making use of Kalman filter algorithms and least square methods to calculate the orbits of low orbit spacecraft. In conjunction with this, imitative simulations are carried out.

1 ALGORITHMS FOR SELECTING SATELLITES

As far as making use of GPS to carry out navigational positioning is concerned, users normally want to select four or more navigational satellites. The relative geometrical positions of these four navigational satellites has very great influences on navigational positioning accuracy. When ranging errors are fair sized, positioning errors produced by different geometrical relationships between users and these four satellites are also different. As a result, selection should be made of satellite constellations associated with optimum geometrical positions in order to obtain the best positioning accuracy.

Up to the present time, GPS system satellite constellation selection methods which relevent references have introduced can be divided into the four types below [2]: optimum geometrical/23 precision factor method [3], maximum vector terminal tetrahedron volume method [4], maximum orthogonal projection method [5], combination method [6]. These four types of methods each have their respective characteristics. With regard to ground or low altitude targets (aircraft, and so on), all of them are capable of obtaining relatively good results. However, speaking in regard to low orbit spacecraft several hundred thousand meters off the ground--due to limitations in the radiation angles of antennas on GPS navigation satellites (< ± 14.3°)--user receiver antenna directional patterns are not complete either (generally smaller than ±80°). Besides this, there are also such problems as the blocking of solar energy battery sail panels, and so on.

As a result, the selection of navigation satellites still has some special spots to it. Below, a usability analysis is carried out.

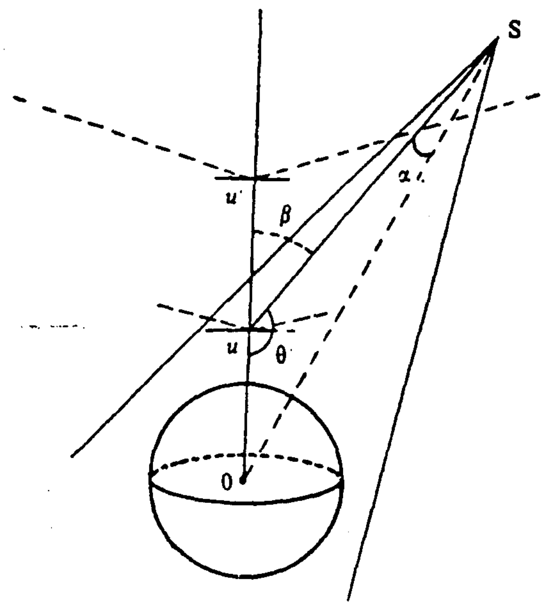

Fig.1 "Usable" Navigation Satellite Schematic

GPS navigation satellites are 20182km away from the ground. Due to limitations on their antenna radiation angles, and user receiver antenna directional pattern angles smaller than ±80°, as a result, when low orbit spacecraft are flying at a certain altitude in space, there is a possibility to create "visible" and not "usable" situations. That is, on the basis of receiver antenna directional patterns, navigation satellites are indeed located within the receiving range, but the transmited beams of navigation satellites are still not able to cover user receiving antennas. Because of this, the navigation satellites in question are then not "usable". Navigation satellites which can be used to supply user positioning should possess two conditions--they must be "visible", and they must also be "usable". As is shown in Fig.1, when $\beta \leq 80°$, navigation satellites are visible. When $a \leq 143°$, navigation satellites are usable. When the geometrical positions of navigation satellites and users simultaneously satisfy the two conditions above, can just be used in positioning. In Fig.1, satellite S is usable and visible with

respect to u. However, it is visible but not usable with regard to u'.

After precisely determining usable satellites, it is then possible to opt for the use of various types of methods to calculate GDOP values in order to precisely specify four navigation satellites used for navigational positioning.

2 DATA PREPROCESSING

GPS systems are passive ranging systems. Users go through measurements of their own relative to already known positions, speeds, and clock differences associated with pseudo ranges and pseudo range rates of the four navigation satellites to precisely determine three dimensional coordinates, speeds, and clock differences of user clocks relative to GPS time. On the basis of analysis, one has:

$$\sigma_p = \sigma \cdot G_{DOP} \tag{1}$$

In this, σp is positioning error;
σ is the mean square deviation associated with pseudo range measurements;
GDOP is the geometrical error factor.

It is possible to see that positioning errors are dependent on geometrical error factors as well as the precision of pseudo range measurements. In order to increase the precision of ranging, on the one hand--in equipment--it is possible to opt for the use of various types of methods. On the other hand--in terms of data processing--it is possible, by contrast, to opt for the use of data smoothing methods in order to reduce ranging errors carried along with random noise.

With respect to internal reciever noise as well as exterior interference, pseudo ranges obtained in measurements can appear strange, that is, "wild values". If preprocessing is not gone through and direct use is made in positioning calculations, relatively large errors will be produced. This article opts for the use of conversion methods to carry out preprocessing with regard to measurement values.

First of all, a conversion time period interval (normally 1s) is selected. Each pseudo range measurement value Sn(tj) within this time interval is taken (in this, tH ≤ tj ≤ tk , tk is the interval end point) and converted to interval end /24

$$S_n(t_j, t_k) = S_n(t_j) + \frac{\lambda}{2\pi}\Delta\psi(t_j, t_k) \tag{2}$$

In this λ -- carrier wave wave length;
$\Delta\psi n$ (tj,tk) -- time interval carrier wave phase increment from tj to tk .

After that, the mean pseudo range value is calculated out for instant tk

$$\bar{S}_n(t_k) = \frac{1}{M}\sum_{t_j=t_H}^{t_j=t_k} S_n(t_j, t_k) \tag{3}$$

As far as application of this type of method is concerned, it is possible--to a very great extent--to restrain the influences of abnormal values, thereby increasing positioning calculation accuracy.

3 USE OF LEAST SQUARE METHODS IN ORBIT DETERMINATION

Processing of observation data is one important link associated with measurement operations. Traditional orbit determination methods make use of least square principles. After waiting for completion of observation sampling, use is made of batch processing methods to calculate the orbit associated with a certain epoch instant. This is also nothing else than the orbital improvement which is often spoken of.

psuedo range

$$\left.\begin{aligned} r_i^* &= r_i + c \cdot b \\ &= [(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2]^{1/2} + c \cdot b \\ \dot{r}_i^* &= \frac{(x - x_s)(\dot{x} - \dot{x}_s) + (y - y_s)(\dot{y} - \dot{y}_s) + (z - z_s)(\dot{z} - \dot{z}_s)}{[(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2]^{1/2}} + c \cdot f \end{aligned}\right\} \tag{4}$$

pseudo range rate

In this, x,y,z are three dimensional user coordinates;

$\dot{x},\dot{y},\dot{z}$ are three dimensional user velocities;
xs,ys,zs are three dimensional positions of navigation satellites;
$\dot{x}_s,\dot{y}_s,\dot{z}_s$ are three dimensional velocities of navigation satellites;
b is user clock deviation;
f is user frequency deviation;
c is the speed of light.

In a certain estimated user state, a Taylor expansion is carried out

$$
\begin{aligned}
r_i^{*} &= f\ (x,\ y,\ z,\ \dot{x},\ \dot{y},\ \dot{z},\ b,\ f) \\
&= r_{io} + r_i^{*'} \cdot \triangle x + o\ (\triangle x^{2}) \\
&= D_i + \frac{\partial r_1^{*}}{\partial x} \cdot \triangle x + \frac{\partial r_1^{*}}{\partial y} \cdot \triangle y + \frac{\partial r_1^{*}}{\partial z} \cdot \triangle z + \frac{\partial r_1^{*}}{\partial x} \cdot \triangle \dot{x} + \frac{\partial r_1^{*}}{\partial y} \cdot \triangle \dot{y} + \frac{\partial r_1^{*}}{\partial z} \cdot \triangle \dot{z} + \\
&\quad \frac{\partial r_1^{*}}{\partial b} \cdot \triangle b + \frac{\partial r_1^{*}}{\partial f} \cdot \triangle f + o\ (\triangle x^{2})
\end{aligned}
$$

In these equations, o(△x2) is a high order term.

After omiting high order terms, the matrix is written in the form of /25

$$
\underset{L_i}{\begin{bmatrix} r_1^{*} - D_1 \\ r_2^{*} - D_2 \\ \vdots \\ \vdots \\ \vdots \\ r_n^{*} - D_n \end{bmatrix}} = \underset{A_i}{\begin{bmatrix} \frac{\partial r_1^{*}}{\partial x} & \frac{\partial r_1^{*}}{\partial y} & \frac{\partial r_1^{*}}{\partial z} & \frac{\partial r_1^{*}}{\partial x} & \frac{\partial r_1^{*}}{\partial y} & \frac{\partial r_1^{*}}{\partial z} & \frac{\partial r_1^{*}}{\partial b} & \frac{\partial r_1^{*}}{\partial f} \\ \vdots & & & & & & & \\ \vdots & & & & & & & \\ \vdots & & & & & & & \\ \frac{\partial r_n^{*}}{\partial x} & \frac{\partial r_n^{*}}{\partial y} & \frac{\partial r_n^{*}}{\partial z} & \frac{\partial r_n^{*}}{\partial x} & \frac{\partial r_n^{*}}{\partial y} & \frac{\partial r_n^{*}}{\partial z} & \frac{\partial r_n^{*}}{\partial b} & \frac{\partial r_n^{*}}{\partial f} \end{bmatrix}} \underset{\triangle X}{\begin{bmatrix} \triangle x \\ \triangle y \\ \triangle z \\ \triangle \dot{x} \\ \triangle \dot{y} \\ \triangle \dot{z} \\ \triangle b \\ \triangle f \end{bmatrix}} \qquad (5)
$$

In this, n is an integer associated with pseudo range;

rio = Di is the range value from the user to the ith navigation satellite (i = 1,2,...,n) for instant k;

Di = [(xsi - x)2 + (ysi - y) + (zsi - z)2]1/2 ;

x, y, z are user coordinates calculated for instant k;

$$\frac{\partial \dot{r}_1^*}{\partial x} = \frac{x_{si} - x}{r_i}; \quad \frac{\partial \dot{r}_1^*}{\partial y} = \frac{y_{si} - y}{r_i}; \quad \frac{\partial \dot{r}_1^*}{\partial z} = \frac{z_{si} - z}{r_i};$$

$$\frac{\partial \dot{r}_1^*}{\partial \dot{x}} = \frac{\partial \dot{r}_1^*}{\partial \dot{y}} = \frac{\partial \dot{r}_1^*}{\partial \dot{z}} = \frac{\partial \dot{r}_1^*}{\partial f} = 0; \quad \frac{\partial \dot{r}_i^*}{\partial b} = c.$$

$$r_i = \sqrt{(x - x_{si})^2 + (y - y_{si})^2 + (z - z_{si})^2}$$

(6)

In the same way, taking $\dot{r}_i^*$ and also carrying out a Taylor expansion under this estimated state

$$\begin{aligned}
\dot{r}_i^* &= g\ (x,\ y,\ z,\ \dot{x},\ \dot{y},\ \dot{z},\ b,\ f) \\
&= \dot{r}_{is} + \dot{r}_i^* \cdot \triangle X + o\ (\triangle x^2) \\
&= \dot{D}_i + \frac{\partial \dot{r}_1^*}{\partial X} \cdot \triangle x + \frac{\partial \dot{r}_1^*}{\partial y} \cdot \triangle y + \frac{\partial \dot{r}_1^*}{\partial z} \cdot \triangle z + \frac{\partial \dot{r}_1^*}{\partial \dot{x}} \cdot \triangle \dot{x} + \frac{\partial \dot{r}_1^*}{\partial \dot{y}} \cdot \triangle \dot{y} + \frac{\partial \dot{r}_1^*}{\partial \dot{z}} \cdot \triangle \dot{z} + \\
&\quad \frac{\partial \dot{r}_1^*}{\partial b} \cdot \triangle b + \frac{\partial \dot{r}_1^*}{\partial f} \cdot \triangle f + o\ (\triangle X^2)
\end{aligned}$$

After omiting high order terms, the matrix is written in the form of

$$\underbrace{\begin{bmatrix} \dot{r}_1^* - \dot{D}_1 \\ \dot{r}_2^* - \dot{D}_2 \\ \vdots \\ \vdots \\ \vdots \\ \dot{r}_n^* - \dot{D}_n \end{bmatrix}}_{L_i} = \underbrace{\begin{bmatrix} \frac{\partial \dot{r}_1^*}{\partial x} & \frac{\partial \dot{r}_1^*}{\partial y} & \frac{\partial \dot{r}_1^*}{\partial z} & \frac{\partial \dot{r}_1^*}{\partial \dot{x}} & \frac{\partial \dot{r}_1^*}{\partial \dot{y}} & \frac{\partial \dot{r}_1^*}{\partial \dot{z}} & \frac{\partial \dot{r}_1^*}{\partial b} & \frac{\partial \dot{r}_1^*}{\partial f} \\ \vdots & & & & & & & \\ \vdots & & & & & & & \\ \vdots & & & & & & & \\ \frac{\partial \dot{r}_n^*}{\partial x} & \frac{\partial \dot{r}_n^*}{\partial y} & \frac{\partial \dot{r}_n^*}{\partial z} & \frac{\partial \dot{r}_n^*}{\partial \dot{x}} & \frac{\partial \dot{r}_n^*}{\partial \dot{y}} & \frac{\partial \dot{r}_n^*}{\partial \dot{z}} & \frac{\partial \dot{r}_n^*}{\partial b} & \frac{\partial \dot{r}_n^*}{\partial f} \end{bmatrix}}_{A_i} \underbrace{\begin{bmatrix} \triangle x \\ \triangle y \\ \triangle z \\ \triangle \dot{x} \\ \triangle \dot{y} \\ \triangle \dot{z} \\ \triangle b \\ \triangle f \end{bmatrix}}_{\triangle X} \quad (7)$$

In this, n is an integer associated with pseudo speed;

$\dot{r}_{is} = \dot{D}_i$ which is the relative motion velocity calculated for instant k between the user and the ith navigation satellite;

$$D_i = \frac{(x_{si} - x)(\dot{x}_{si} - \dot{x}) + (y_{si} - y)(\dot{y}_{si} - \dot{y}) + (z_{si} - z)(\dot{z}_{si} - \dot{z})}{[(x_{si} - x)^2 + (y_{si} - y)^2 + (z_{si} - z)^2]^{1/2}};$$

;

x,y,z are three dimensional user coordinates; /26

$\dot{x}, \dot{y}, \dot{z}$ are three dimensional user velocities;

$$\frac{\partial \dot{r}_i}{\partial x} = \frac{x - x_{si}}{r_i} + [(x - x_{si})(\dot{x} - \dot{x}_{si}) + (y - y_{si})(\dot{y} - \dot{y}_{si}) + (z - z_{si})(\dot{z} - \dot{z}_{si})\frac{\partial r_i}{\partial x};$$

$$\frac{\partial \dot{r}_i}{\partial y} = \frac{y - y_{si}}{r_i} + [(x - x_{si})(\dot{x} - \dot{x}_{si}) + (y - y_{si})(\dot{y} - \dot{y}_{si}) + (z - z_{si})(\dot{z} - \dot{z}_{si})\frac{\partial r_i}{\partial y};$$

$$\frac{\partial \dot{r}_i}{\partial z} = \frac{z - z_{si}}{r_i} + [(x - x_{si})(\dot{x} - \dot{x}_{si}) + (y - y_{si})(\dot{y} - \dot{y}_{si}) + (z - z_{si})(\dot{z} - \dot{z}_{si})\frac{\partial r_i}{\partial z};$$

$$\frac{\partial \dot{r}_i}{\partial \dot{x}} = \frac{x - x_{si}}{r_i}; \frac{\partial \dot{r}_i}{\partial \dot{y}} = \frac{y - y_{si}}{r_i}; \frac{\partial \dot{r}_i}{\partial \dot{z}} = \frac{z - z_{si}}{r_i}; \frac{\partial \dot{r}_i}{\partial b} = 0; \frac{\partial \dot{r}_i}{\partial f} = c.$$

On the basis of formula (4) and formula (6), one also has

$$\frac{\partial r_1}{\partial x} = \frac{x_{si} - x}{r_i} = \frac{\partial \dot{r}_1}{\partial \dot{x}}$$

$$\frac{\partial r_1}{\partial y} = \frac{y_{si} - y}{r_i} = \frac{\partial \dot{r}_1}{\partial \dot{y}}$$

$$\frac{\partial \dot{r}_1}{\partial z} = \frac{z_{si} - z}{r_i} = \frac{\partial \dot{r}_1}{\partial \dot{z}}$$

Merging the two matrices into one, one obtains

$$L = \begin{bmatrix} L_r \\ L_i \end{bmatrix} = \begin{bmatrix} A_r \\ A_i \end{bmatrix} \cdot \triangle X = A \cdot \triangle X \tag{8}$$

In the equations, L is the observation matrix; A is the gradient matrix; Δ X is the state vector correction amount.

From matrix theory, it is possible to know that the equation A ,Δ X = L's least square solution is

$$\Delta X = (ATA)-1AT . L$$

From this, it is possible--on the basis of the values of each iteration of measurements--to calculate user state deviation corrections to carry out calibrations with regard to the next state, Xk+1 = Xk + ΔX. After that, new calculations are done of Di and $\dot{D}_i$, making use of new observation quantities to calculate L matrices, repeating the corrections above. In this way, after going through a certain number of iterative substitutions, it is then possible to precisely determine user positions, speeds, and other condition values. In the calculations discussed above, after each iterative substitution, new calculations should be done of coefficient matices A and observation matrices L.

4 KALMAN WAVE FILTER CALCULATIONS AND THEIR REALIZATION

Classical Kalman wave filtering is linear least square variance wave filtering. When systems satisfy the conditions below, only then are wave filter solutions optimal.

(1) System dynamics models and measurement models are both linear;

(2) System dynamics models and meaurement models match up with reality;

(3) Initial state conditions and a priori statistical characteristics associated with noise models are both zero mean value distributions of already known variances.

If the conditions discussed above are not satisfied, the results obtained will only be capable of being approximately optimal. Due to the fact that the majority of engineering problems are associated with dynamic systems and measurement /27 systems which are, in and of themselves, not linear, as a result, there is a need to opt for the use of generalized Kalman wave filtering.

Assume the continuous nonlinear dynamic equation is

$$\frac{\partial X(t)}{\partial t} = f(X(t)) + W_t \tag{9}$$

In equations, X(t) is the n dimensional state vector; Wt is the m dimensional model noise vector. It is zero mean value Gaussian white noise. Moreover,

$$\left.\begin{aligned} &E\{W_t\} = 0 \\ &E\{W_t \cdot W_{t'}^T\} = Q(t)\delta(t-t') \end{aligned}\right\} \tag{10}$$

In equations, Q(t) is the state noise matrix.

Assume again that the measurement equation is the dispersion nonlinear equation

$$Zk+1 = H(Xk+1)\ Vk+1$$

In equations, Zk+1 represents observation vectors at instant k+1. Vk+1 is measurement noise. It is zero mean value Gaussian white noise

$$\left.\begin{array}{l} E\{V_k\} = 0 \\ E\{V_k \cdot V_l^T\} = R_k \delta_{kl} \end{array}\right\} \tag{11}$$

In equations, Rk a measurement noise matrix.

Wave filter recurrence equations are [7]:

(1) Calculation Prediction Values

$$\hat{X}_{k+1|k} = \hat{X}_{k|k} + f(\hat{X}_{k|k})T + A(\hat{X}_{k|k})f(\hat{X}_{k|k})\frac{T^2}{2} \tag{12}$$

In this, $A(x) = \frac{\partial f(x)}{\partial x}$; T is the sampling interval.

(2) Calculation Prediction Error Variance Matrix

$$P_{k+1|k} = \Phi_k P_{k|k} \Phi_k^T + Q_K \tag{13}$$

In this, $\Phi_k = I + A(\hat{X}_{k|k})T$

(3) Calculation Gain Matrix

$$K_{k+1} = P_{k+1|k} B_{k+1}^T (B_{k+1} P_{k+1|k} B_{k+1}^T + R_{k+1})^{-1} \tag{14}$$

In this, $B_{K+1} = B(\hat{X}_{k+1|k})$

$$B(x) = \frac{\partial H(X)}{\partial X}$$

(4) Calculation Wave Filter Error Variance Array

$$P_{k+1|k+1} = [I - K_{k+1} B_{k+1}] P_{k+1|k} \tag{15}$$

(5) Calculation Wave Filter Values

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1}[Z_{k+1} - H(\hat{X}_{k+1|k})] \tag{16}$$

Initial recurrence values $X_{0|0}, P_{0|0}$

As far as the use of Kalman wave filter algorithms in orbit determinations is concerned, consideration should be given to the several questions below.

(1) Selection of State Variables

Speaking of state variable selection with regard to Kalman wave filtering is an important question. Three dimensional user positons and velocities are, of course, state variables. Due to the fact that GPS systems make use of user clocks in order to determine electric wave arrival times, converting them into ranging, and thereby carrying out positioning, clock differences and frequency differences will directly bring with them errors in ranging and speed determinations. As a result, user clock differences and frequency differences will also act as state variables. Speaking with regard to low orbit spacecraft, the principal perturbation forces influencing their orbits are the gravitational forces associated with the nonspherical shape of the earth and atmospheric drag. Moreover, comparing various other types of perturbation forces to these two, they are all high order and small in amount. They can be regarded completely as model noise. As far as gravitational forces associated /28 with the nonspherical nature of the earth are concerned, it is possible to make use of mathematical models to compare and accurately describe them. However, atmospheric drag, by contrast, changes continuously due to spacecraft attitudes. As a result, in order to accurately determine spacecraft positions and velocities, aerodynamic factors should be regarded as a state variable. In this way, 9 state variables are included in wave filter equations.

$$X = [x, y, z, \dot{x}, \dot{y}, \dot{z}, b, f, d]^T$$

(2) Dynamics Models Associated with Wave Filter Devices

Satellite motion equations are

$$\left.\begin{aligned} \ddot{x} &= \frac{\partial U}{\partial x} \\ \ddot{y} &= \frac{\partial U}{\partial y} \\ \ddot{z} &= \frac{\partial U}{\partial z} \end{aligned}\right\} \qquad (17)$$

In these, U = (GM/r) + R; GM is the gravitational constant of the earth (3986005x108m3/s2) ; r is the length from the coordinate origin point to the center of mass of the user; R is perturbation functions.

In this, only giving consideration to gravitational forces

associated with the nonspherical shape of the earth and atmospheric drag, spacecraft motion equations can then be written as

$$\left.\begin{aligned}\ddot{x} &= -\frac{GM}{r^3}x + A_{1x} + A_{2x}\\ \ddot{y} &= -\frac{GM}{r^3}y + A_{1y} + A_{2y}\\ \ddot{z} &= -\frac{GM}{r^3}z + A_{1z} + A_{2z}\end{aligned}\right\} \qquad (18)$$

In this, A1x , A1y , and A1z are three directional accelerations given rise to by perturbations in gravitational forces associated with the earth; A2x, A2y , and A2z are three directional accelerations given rise to by atmospheric drag perturbations.

(1) Gravitational Perturbation Acceleration A1 Associated with the Nonspherical Shape of the Earth

The potential function associated with the earth is [8]

$$U = \frac{GM}{r} - \frac{GM}{r}\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\Phi)J_{nm}\cos m(\lambda - \lambda_{nm}) \qquad (19)$$

In this, $\Phi = \operatorname{arctg}\frac{z}{(x^2+y^2)^2}$;

m is satellite mass;
λ is satellite geographical longitude;
ae is WGS-84 ellipsoid hemi major axis (6378137m)[10];
Jnm and λnm are harmonic coefficients.

Going through derivations and appropriate choices, perturbation accelerations associated with the earth's gravitational forces are

$$\left.\begin{aligned}A_{1x} &= -GM\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{3x}{2r^3} - \frac{15x}{2r^3}\sin\Phi\right]\\ A_{1y} &= -GM\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{3y}{2r^3} - \frac{15y}{2r^3}\sin\Phi\right]\\ A_{1z} &= -GM\left(\frac{a_e}{r}\right)^2 J_{20}\left[\frac{3}{2r^2}\sin\Phi - \frac{15}{2r^2}\sin^3\Phi\right]\end{aligned}\right\} \qquad (20)$$

In this, J20 is the gravitational force field second order band harmonic term 10.8263x10-8 . /29

(2) Atmospheric Drag Perturbation Acceleration A2

Atmospheric drag perturbation acclerations are expressed as

$$\vec{A}_2 = \frac{C_D A \rho}{2m} V'^2 \left(- \frac{\vec{V}'}{V'} \right) = d \cdot \rho \cdot V'^2 \left(- \frac{\vec{V}'}{V} \right) \tag{21}$$

In this d is aerodynamic factors;
ρ is atmospheric density;
$\vec{V}'$ is satellite motion speed relative to the atmosphere.

Going through resolution, it is possible to obtain

$$\left. \begin{aligned} A_{2x} &= -\rho V'(\dot{x} + y w_e) d \\ A_{2y} &= -\rho V'(\dot{y} - x w_e) d \\ A_{2z} &= -\rho V' \dot{z} d \end{aligned} \right\} \tag{22}$$

The analysis discussed above is based on an execution within an inertial coordinate system. However, the WGS-84 coordinate system which the GPS system opts for the use of is an earth fixed coordinate system. As a result, it is necessary to carry out the transformations

$$\left. \begin{aligned} \ddot{x} &= -\frac{G}{r^3} x + A_{1x} + A_{2x} + w_e^2 x + 2 w_e \dot{y} \\ \ddot{y} &= -\frac{G}{r^3} y + A_{1y} + A_{2y} + w_e^2 y - 2 w_e \dot{y} \\ \ddot{z} &= -\frac{G}{r^3} z + A_{1z} + A_{2z} \end{aligned} \right\} \tag{23}$$

(3) Clock and Aerodynamic Factor Models

On the basis of dynamics equations--using clock deviation b, frequency deviation f, and aerodynamic factor d as state variables--systems should satisfy

$$\left. \begin{aligned} \dot{b} &= f + w_b \\ \dot{f} &= -\frac{1}{\tau_f} f + w_f \\ \dot{d} &= -\frac{1}{\tau_d} d + w_d \end{aligned} \right\} \tag{24}$$

In this, τf and τd are first order Maerkefu (phonetic) process correlation times. wb , wf , and wd are all Gaussian white noise.

(4) Nine State Wave Filter Device Dynamics Model

$$X = [x, y, z, \dot{x}, \dot{y}, \dot{z}, b, f, d]^T$$
$$\dot{X} = f[X(t)] + W$$

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$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -\frac{GM}{r^3}x + A_{1x} - \rho V(\dot{x} + y\omega_e)d + \omega_e^2 x + 2\omega_e \dot{y} \\ -\frac{GM}{r^3}y + A_{1y} - \rho V(\dot{y} - x\omega_e)d + \omega_e^2 y + 2\omega_e \dot{x} \\ -\frac{GM}{r^3}z + A_{1z} - \rho V \dot{z} d \\ f \\ -\frac{1}{\tau_f}f \\ -\frac{1}{\tau_d}d \end{bmatrix} + \begin{bmatrix} W_x \\ W_y \\ W_z \\ W_{vx} \\ W_{vy} \\ W_{vz} \\ W_b \\ W_f \\ W_d \end{bmatrix}$$

(5) Precise Determination of Model Noise Variance Matrices Q

Seen from the view of general references, as far as model noise variance matices Q are concerned, it is possible to select appropriate values--on the basis of problems--relying on experience and experimentation. Among those commonly used are the several types discussed below [9]:

-- opting for the use of rigorous dynamics models in order to do calculations of state vectors;
-- going through simulation calculations to precisely determine relatively appropiate Q;
-- going through calculation results as a whole to precisely determine and, in conjunction with that, adjust Q, making differences between Kalman wave filters relatively small and selecting Q for this time to function as dynamic noise covariance;
-- with regard to maneuvering targets, opting for the use of self-adjusting wave filters.

The influences of dymanics model errors on orbits are primarily dependent on the length of orbit segmental arcs. The errors associated with long segmental arc orbits primarily come from dynamics model uncertainties. Speaking in terms of short

segmental arcs, the influences of dynamics model errors are relatively small, and it is primarily observation noise which influences orbit results. As a result--speaking with regard to low orbit spacecraft orbits which are relatively stable--Q can be selected as a constant value.

(3) Precise Determination of Measurement Equations and Acutalization of Generalized Kalman Wave Filter Devices

GPS goes through the carrying out of measurements of psuedo ranges and pseudo range speeds with regard to four navigation satellites in order to calculate user positions and velocities. Thus, the observed quantities are pseudo ranges and pseudo range velocities. Moreover, these two measured values are nonlinear with regard to state variables $x、y、z、\dot{x}、\dot{y}、\dot{z}$. For this reason, they should be used to take a certain nominal state expansion and turn it into a Taylor series. Omiting higher than second order terms, a linear measurement equation is constructed. Moreover, nominal states can be selected to be $\hat{X}_{k+1|k}$. Linearization is carried out respectively for pseudo ranges and pseudo range velocities in regard to the four satellites, thereby constructing linearized measurement equations. Having wave filter device state models and measurement models, it is then possible to realize Kalman wave filtering. In conjunction with this, calculations of spacecraft orbits are carried out.

5 SIMULATION

This article opts for the use of Monte Carlo methods to carry out orbital determination calculation simulations. Results clearly show that--when one possesses real time observation data--use is made of minimum square methods, and it is possible to obtain relatively high positioning accuracies--σr = 15.587m, σv = 0.056m/s, σt = 0.001ms. Making use of Kalman wave filter methods, it is possible, in the same way, to achieve relatively high accuracies--σr = 17.048m, σv = 0.048m/s, σt = 0.0025ms (These simulation results do not consider the influences of electric wave propogation nor do they consider SA influences).

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